EDUCATION MIND

VOL 5, NO 1, 1-16
DOI: 10.58583/EM.4.1.1

Research Article

Open Access

Identifying misconceptions of 5th grade students regarding triangles and quadrilaterals

Beyza Macit [1], **Sanem Tabak** [1]

[1] Department of Educational Sciences, Ordu University, Ordu, Türkiye

ABSTRACT

This study aimed to identify fifth-grade students' misconceptions about triangles and quadrilaterals. A two-tier diagnostic test developed by the researchers was used to identify students' misconceptions. The content validity of the test questions was assessed using the Davis Technique form and expert's opinion; validity, reliability, and discriminative properties were analyzed using the TAP through a pilot application with 52 participants. The main applications of the research were conducted with 204 students in four different schools in the province of Ordu. According to the research results, misconceptions were identified in the types of overgeneralization, overspecification, misinterpretation, and limited perception regarding naming polygons, creating them, and defining their basic elements. It was found that students had misconceptions in the types of overgeneralization and limited perception regarding creating triangles based on their angles and sides and classifying different triangles based on their side and angle properties. It was determined that 5th grade students had misconceptions in the types of overgeneralization, mistranslation, and restricted perception regarding identifying and drawing the basic elements of rectangles, parallelograms, rhombuses, and trapezoids. It was determined that students only had misconceptions of the type of limited perception regarding determining the sum of the interior angles of triangles and quadrilaterals and finding the unknown angle.

ARTICLE HISTORY

Received 28 February 2026
Accepted 04 April 2026

KEYWORDS

Concept
Misconception
Two-tier diagnostic test
Triangles and quadrilaterals

CORRESPONDENCE

Sanem Tabak

Introduction

Mathematics is systematic and rule-based thinking that establishes mental connections in accordance with logic (Altun, 2018). Mathematics education aims to develop logical thinking skills and to understand existing situations by connecting them to real-life experiences, thereby imparting problem-solving skills (Altun, 2018; Dağ, 2022).

Geometry is a branch of science that establishes logical reasoning regarding the shape, dimensional, and conceptual aspects of the universe and all entities within it (Yurtyapan & Karataş, 2020). According to the curriculum in Turkey, geometry education begins at the preschool level and continues throughout the student's educational life. The main geometry topics covered in the elementary school mathematics curriculum are listed below (MoNE, 2018):

- At the elementary school level: geometric objects and shapes, spatial relations, geometric patterns, and basic concepts in geometry.
- At the middle school level: basic geometric concepts and drawings, triangles and quadrilaterals, measuring length and time, measuring area, geometric objects, lines and angles, circles and spheres, measuring liquids, transformation geometry, polygons, congruence and similarity.

The ‘concept’, which is most frequently addressed in the curriculum and regarded as the cornerstone of learning or thinking, is the grouping of objects or events under a common heading (Gökdoğan, 2021; Öksüz & Başışık, 2019). Individuals learn concepts throughout their lives, go to school with certain concepts they have learned, and continue to learn concepts at school (Öksüz & Başışık, 2019). Indeed, concepts are included under the subject headings in the learning outcomes section of teaching programs (MoNE, 2018). While acquiring a concept, it is beneficial to carefully process the stages of acquisition; however, if these concepts are misconstructed, they are considered erroneous or incorrect and are labeled as misconceptions because they contradict scientific facts (Gökdoğan, 2021).

Misconceptions are defined as concepts that are incorrectly or erroneously structured, as well as concepts that are structured in opposition to scientific facts (Başışık, 2010). From a learning perspective, they can be described as abstractions made in the mind as a result of learning that contradict scientific standards (Hacısalihoğlu-Karadeniz & Hodancı, 2022). In the literature, misconceptions are also referred to as initial perceptions, alternative perceptions, preconceptions, and unformed perceptions. The common point among all these names and definitions is perception, which can be said to be the source of conceptual misconceptions (Zembat, 2008).

Internal and external factors influencing individuals’ perceptions lead to various types of conceptual misunderstandings. The first of these, overgeneralization, involves applying judgments about a concept to similar situations that fall outside its scope (Zodik & Zaslavsky, 2008). In contrast, over-specification refers to the restriction of a concept to only a specific prototypical feature (Clements & Battista, 1992). Mistranslation involves the manipulation of meaning, particularly during the process of mixing mathematical language with everyday language (Pimm, 1987). Finally, limited perception refers to a student creating an incomplete model of the concept by referring only to a part of the definition (Fischbein, 1993).

Research on the causes of conceptual misunderstandings offers various perspectives. In the literature, these causes are classified as: preconceptions acquired from the environment (Smith, diSessa & Roschelle, 1993), the mismatch between the formal language used in the teaching process and mental images (Vinner, 1991), and the failure to sufficiently concretize abstract concepts (Hiebert & Carpenter, 1992). Furthermore, the individual’s prior knowledge and past experiences, when they conflict with current scientific models, create a complex network of interactions in the formation of misconceptions (Vosniadou, 1994).

Conceptual misunderstandings encountered in geometry instruction are not merely a matter of terminological deficiency but also a problem of cognitive structuring. Tall and Vinner (1981) explain this situation as a disconnect between the “concept image” and the “concept definition.” When all the visual and mental associations (images) formed in a student’s mind regarding a concept (such as a triangle) do not align with the concept’s formal academic definition, a conceptual misunderstanding becomes inevitable. According to the Van Hiele (1986) model, which explains the development of geometric thinking, students’ failure to transition from the visual level (Level 0) to the analytical level (Level 1) prevents them from grasping the critical properties of shapes. In this context, the “prototype effect” (Hershkowitz, 1990), frequently emphasized in the literature, causes students to recognize only shapes in standard orientations (base parallel to the ground, symmetrical) while excluding variations beyond these.

Mathematics is a field of science that individuals encounter in their academic lives. Therefore, individuals encounter many concepts while learning mathematics. At the same time, mathematics has a spiral structure, and the misconceptions individuals have at the beginning cause new misconceptions in the learning of new topics (Dağ, 2022). Therefore, it is important to identify and eliminate the mathematical misconceptions that students have. Extensive research has also been conducted on rational numbers (Behr et al., 1983; Vamvakoussi & Vosniadou, 2004), algebraic expressions (Booth, 1988), and geometric concepts (Clements & Battista, 1992). These studies demonstrate that conceptual misunderstandings are not merely a local issue but rather a reflection of the universal cognitive stages students go through as they construct mathematical structures in their minds. Specifically in Turkey, there are studies in mathematics education that identify students' misconceptions (Araz, 2024; Hacısalihoğlu-Karadeniz & Hodancı, 2022; Kabataş, 2022; Öksüz & Başışık, 2019; Sarı, 2024; Şahiner, 2018). These studies cover integers (Hacısalihoğlu-Karadeniz & Hodancı, 2022); natural numbers (Kabataş, 2022); decimal numbers (Sarı, 2024); rational numbers (Araz, 2024); algebraic expressions (Şahiner, 2018). Also, Türkdoğan et al. (2015) reviewed studies conducted on misconceptions in mathematics education in Turkey between 1999 and 2013. The studies found that mathematical misconceptions arise as a result of students learning incorrectly or incompletely and that concepts with similar names, properties, or functions are confused. According to the research results, it was determined that the studies were mostly aimed at identifying mathematical misconceptions; the studies were mostly conducted on middle school and high school students; and the studies focused on identifying misconceptions in topics related to learning numbers and operations. There are a limited number of studies that reveal students' misconceptions about triangles and quadrilaterals, which are the basis of the present study. In the study by Kemankaşlı and Gür (2005), which identified the mistakes of 11th-grade students on the topic of quadrilaterals, it was determined that students made mistakes when establishing the side-angle relationship in special triangles and rectangles based on their angles and could not grasp the properties of parallelograms and rhombuses. Başışık (2010) conducted a study to identify misconceptions among 5th grade students regarding polygons and quadrilaterals. The study identified misconceptions such as having a distorted perception of prototype shapes, believing that the diagonal of a triangle is a hypotenuse, and believing that irregular quadrilaterals are not polygons. Baran (2011) examined the misconceptions of 6th, 7th, and 8th grade students regarding triangles and geometric objects; and found that students had misconceptions about adjacent angles, the definition of the hypotenuse, calculating the area of a triangle, classifying types of triangles, distinguishing geometric figures, and the fact that when a triangle is bisected, the sum of its interior angles is 90 degrees. Doğan and colleagues investigated the misconceptions of 6th, 7th, and 8th grade students regarding the concept of a trapezoid; they identified misconceptions that the sides and angles of a trapezoid are not equal, that quadrilaterals with no parallel sides are trapezoids, the prototype shape misconception, and that the sides of a trapezoid are curved. In Ay and Başbay's (2017) study identifying 7th grade students' misconceptions about polygons, it was found that students were unable to identify polygon types and properties, unable to identify concave and convex polygons, unable to understand basic concepts, and had distorted prototype shape perception. Çekiç (2018) examined the misconceptions of 5th grade students regarding basic geometric concepts and drawings. The study identified misconceptions such as confusion about the size of a point, angles, perpendiculars, lines, rays, and line segments, representation with symbols, and thinking that the lengths of line segments are parallel. In Kaya's (2018) study examining 8th grade students' misconceptions in the learning domain of triangles, it was determined that students had misconceptions about the side-angle relationships of triangles, the concepts of sides and vertices, the ordering of angle sizes, triangle drawing, height, angle bisector, and side bisector relationships. As can be seen, there is a limited number of studies specifically on triangles and quadrilaterals in the field of geometry learning, and it is considered important to identify misconceptions in triangles and quadrilaterals, which are one of the most fundamental topics in the teaching of geometry concepts. Furthermore, studies have identified that step-by-step diagnostic tests are rarely used to identify misconceptions. Accordingly, the main objective of this study is to develop a two-stage diagnostic test to identify the misconceptions of 5th grade middle school students regarding "Triangles and Quadrilaterals" and to determine the misconceptions they have on this subject. In this study, the following questions were addressed in line with this fundamental objective:

1. What are the misconceptions of fifth-grade students regarding naming polygons, constructing them, and recognizing their basic elements?
2. What are the misconceptions of fifth-grade students regarding creating triangles based on their angles and sides, and classifying different triangles based on their side and angle properties?
3. What are the misconceptions of fifth-grade students regarding identifying and drawing the basic elements of rectangles, parallelograms, rhombuses, and trapezoids?
4. What are the misconceptions of fifth-grade students regarding determining the sum of the interior angles of triangles and quadrilaterals and finding the unknown angle?

Method

Research design

The study aimed to identify the misconceptions of 5th-grade students regarding triangles and quadrilaterals according to the primary school mathematics curricula (MoNE, 2018). A convergent mixed-methods research design was used in this study. Convergent mixed-methods research is a research design in which quantitative and qualitative data are collected simultaneously and analyzed together (Toraman, 2021). In this study, the "Triangles and Quadrilaterals Two-Tier Diagnostic Test" was developed, and quantitative and qualitative data were collected simultaneously through a test using both multiple-choice and open-ended questions to identify the misconceptions of 5th-grade middle school students. Concurrent mixed methods were chosen as the research design because the study aimed to identify students' misconceptions about triangles and quadrilaterals in detail by collecting quantitative and qualitative data simultaneously and analyzing them together.

Participants

The participants in this study were 5th grade students enrolled in state schools at different achievement levels throughout the province, affiliated with the Ordu Provincial Directorate of National Education during the 2023-2024 academic year. These schools were selected based on different achievement levels using simple random sampling. Studies were conducted with students in the classes of teachers who agreed to participate in the application at five public schools that accepted the application from these schools. During the development of the "Triangles and Quadrilaterals Two-Tier Diagnostic Test" for the research, the process was carried out with 52 fifth-grade students, 26 girls and 26 boys, from one of the five public schools identified for the pilot application within the scope of the research. The research was conducted with 204 students, 90 of whom were girls and 114 of whom were boys, from four public schools to identify the misconceptions of fifth-grade students.

Data collection tool

The "Triangles and Quadrilaterals Two-Stage Diagnostic Test," developed by the researcher as a data collection tool, was used in the study. The following steps were followed in preparing this test and conducting its validity and reliability studies.

While developing the diagnostic test, the two-stage test development steps suggested by Treagust (1988) and Chen, Lin, & Lin (2002) were followed. First, the middle school mathematics teaching program (MoNE, 2018) and the textbooks taught in the 2023-2024 academic year were examined, and propositions were written. A concept map related to the learning outcomes was created. The concept map containing the propositions and learning outcomes was linked. The validity of the propositions and the concept map was checked by an expert in the field of Mathematics Education. A literature review on the subject was conducted. Multiple-choice items were prepared by working on students' possible misconceptions. The second stage of the test was written as open-ended questions. A specification table (Table 1) was prepared for the first stage of the test, and the questions written were ready for expert opinion.

Table 1. Specification Table for the Two-Tier Diagnostic Test

Cognitive Domain / Learning Goals*	Remember	Understand	Apply	Analyze	Create	Evaluate	Total Question
M.5.2.2.1.	10						10
M.5.2.2.2.			2	3			5
M.5.2.2.3.	5						5
M.5.2.2.4.			4				4

* M.5.2.2.1. Names, creates, and identifies the basic elements of polygons.
M.5.2.2.2. Creates triangles based on their angles and sides, and classifies different triangles based on their side and angle properties.
M.5.2.2.3. Identifies the basic elements of rectangles, parallelograms, rhombuses, and trapezoids.
M.5.2.2.4. Determines the sum of the interior angles of triangles and quadrilaterals and finds the unknown angle.
(MoNE, 2018)

Based on the prepared specification table, a 30-item test was developed, with each question consisting of two stages: the first stage being multiple-choice and the second stage containing open-ended questions. Experts' opinions were obtained using the Davis technique to ensure the content validity of the questions included in the 30-item diagnostic test. The Davis technique is a method that determines content validity through expert opinion (Abbak & Gelişli, 2023). Experts were expected to evaluate the test questions using the options "appropriate, needs minor revision, needs major revision, and should be removed." In the Davis technique, the number of experts who marked the "appropriate" and "needs minor revision" options was divided by the total number of experts to calculate the content validity index (CVI) for each item, and the resulting value was expected to be higher than 0.8 (Davis, 1992; cited in Abbak & Gelişli, 2023). Accordingly, information regarding the CVI value for each item in the diagnostic test is provided in Table 2.

Table 2. CVI Values of Items Included in the Test

Items	Appropriate	Needs minor revision	Needs major revision	Should be removed	CVI
1	4	1	0	0	1,00
2	3	2	0	0	1,00
3	5	0	0	0	1,00
4	2	2	0	0	0,80
5	4	1	0	0	1,00
6	3	1	0	1	0,80
7	4	1	0	0	1,00
8	1	3	1	0	0,80
9	4	1	0	0	1,00
10	4	1	0	0	1,00

11	5	0	0	0	1,00
12	5	0	0	0	1,00
13	3	0	0	1	0,60
14	4	1	0	0	1,00
15	3	2	0	0	1,00
16	2	3	0	0	1,00
17	2	3	0	0	1,00
18	3	1	0	1	0,80
19	4	1	0	0	1,00
20	4	0	0	1	0,80
21	1	2	2	0	0,60
22	3	2	0	0	1,00
23	5	0	0	0	1,00
24	5	0	0	0	1,00
25	4	0	0	1	0,80
26	4	1	0	0	1,00
27	3	1	1	0	0,80
28	1	1	1	2	0,40
29	3	2	0	0	1,00
30	3	1	0	1	0,80

Based on the experts' opinions, the CVI values calculated for the 27 items in the test were above 0.80; items 13, 21, and 28 were excluded from the test because their CVI values were below 0.80. Items 2, 4, 6, 8, 15, 16, 17, 18, 22, 27, 28, 29, and 30 in the test were marked by experts as "Needs minor revision," and they expressed their opinions on the correction of these items. Accordingly, the necessary adjustments were made to the relevant items.

While expert opinions were being gathered, a pilot application was conducted with 52 students on 30 questions. The validity and reliability of the questions were analyzed using the TAP programme (Table 3, Table 4).

Table 3. Triangles and Quadrilaterals Two-Tier Diagnostic Test General Analysis Results

N	52
Total Maximum Score	30
Lowest Score	0
Highest Score	26
Median	9,5
Mean	10,404
Standard Deviation	5,013
Variance	25,125
Skewness	0,862
Kurtosis	1,217
Average Difficulty Index of the Test	0,347
Average Discrimination Index of the Test	0,367
KR20	0,787
KR21	0,755
Upper Group Minimum Score(N=14)	13
Lower Group Maximum Score (N=18)	8

When examining Table 3 and Table 4, the maximum score that can be obtained from the diagnostic test based on the overall statistical result is 30; the highest score obtained by students on the test is 26, and the lowest score is zero. The median value of the test is 9.5; the mean is 10.404; the standard deviation value is 5.013; the variance value is 25.125; the skewness value is 0.862; and the kurtosis value is 1.217. When the results of the KR20 and KR21 internal consistency coefficients of the test were examined, values close to 1.00 were found, thus determining that the test is a reliable test.

Table 4. Triangles and Quadrilaterals Two-Tier Diagnostic Test Item Analysis Results

Questions	Number of Correct Answers	Difficulty Index	Discrimination Index	Correct Answers in Upper Group	Correct Answers in Lower Group
1	32	0,62	0,41	12 (0,86)	8 (0,44)

2	30	0,58	0,58	12 (0,86)	5 (0,28)
3	20	0,38	0,42	9 (0,64)	4 (0,22)
4	13	0,25	0,06	4 (0,29)	4 (0,22)
5	28	0,54	0,65	13 (0,93)	5 (0,28)
6*	10	0,19	-0,08	2 (0,14)	4 (0,22)
7	27	0,52	0,52	12 (0,86)	6 (0,33)
8	40	0,77	0,56	14 (1,00)	8 (0,44)
9	21	0,40	0,42	9 (0,64)	4 (0,22)
10	24	0,46	0,58	12 (0,86)	5 (0,28)
11	14	0,27	0,46	8 (0,57)	2 (0,11)
12	19	0,37	0,66	10 (0,71)	1 (0,06)
13*	8	0,15	0,03	2 (0,14)	2 (0,11)
14	16	0,31	0,21	6 (0,43)	4 (0,22)
15	16	0,31	0,21	6 (0,43)	4 (0,22)
16*	8	0,15	-0,17	0 (0,00)	3 (0,17)
17	16	0,31	0,46	8 (0,57)	2 (0,11)
18	11	0,21	0,30	5 (0,36)	1 (0,06)
19	32	0,62	0,60	13 (0,93)	6 (0,33)
20	11	0,21	0,32	6 (0,43)	2 (0,11)
21*	7	0,13	0,23	4 (0,29)	1 (0,06)
22	20	0,38	0,53	9 (0,64)	2 (0,11)
23	14	0,27	0,59	9 (0,64)	1 (0,06)
24	18	0,35	0,40	8 (0,57)	3 (0,17)
25*	22	0,42	0,49	60 (0,71)	4 (0,22)
26	16	0,31	0,42	9 (0,64)	4 (0,22)
27*	10	0,19	0,36	5 (0,36)	0 (0,00)
28*	5	0,10	0,29	4 (0,29)	0 (0,00)
29	21	0,40	0,33	10 (0,71)	7 (0,39)
30	12	0,23	0,17	4 (0,29)	2 (0,11)

When the item analyses of the test were examined, items 6, 13, 21, 25, 27, and 28 were determined to have low validity-reliability analyses and were removed from the test (Table 4). Taking into account expert opinions and item analyses, 6 questions were removed from the test, and the final version of the test was organized as 24 items.

Data analysis

In the study, to identify fifth-grade students' misconceptions, the answers given by students in the first stage of the test were categorized as correct, incorrect, or blank; accordingly, frequency and percentage were obtained from descriptive statistics. The student responses in the second stage of the test were analyzed using content analysis.

When performing content analysis, the questions in the diagnostic test were first categorized according to the research questions. The reasons given by students for their answers to the questions were examined individually and categorized according to the types of misconceptions specified by Zembat (2013) and coded. These types of misconceptions are divided into four categories by Zembat (2013): overgeneralization, overspecification, mistranslation, and limited perception:

- Overgeneralization: Thinking that any feature categorized into a type has the same feature in all closely related categories. An example of this is thinking that all quadrilaterals have equal side lengths, just as all squares have equal side lengths.
- Over-specification: Thinking that any feature categorized by type cannot be a feature of any type other than the existing type. An example of this is thinking that the feature of opposite sides being parallel is only found in parallelograms.
- Incorrect Translation: This is when information is perceived differently from its original meaning when converted into different forms. An example of this is when drawing a right-angled triangle from among the types of triangles, instead of drawing the angle between the sides as a right angle, drawing the height on any triangle and then showing the angle between the line segment indicating the height and the side to which the height belongs as a right angle.
- Limited Perception: This is the incomplete perception of a concept's known definitions. An example of this is knowing that polygons are formed from sides and corners but not knowing/thinking that the sides must be line segments.

In line with the types of misconceptions given above, the first author of the study first identified the misconceptions by examining the students' reasoning. Then, the identified misconceptions were examined with the second author of the study, and both researchers discussed and categorized the identified misconceptions together.

Results

Misconceptions of 5th grade students regarding naming polygons, creating them, and identifying their basic elements

To identify the misconceptions of 5th grade middle school students regarding naming polygons, creating them, and identifying their basic elements, students were asked 10 questions, and their correct, incorrect, and blank answers to these questions are presented in Table 5.

Table 5. Distribution of Students' Answers to Questions

Questions	Correct Answer		Incorrect Answers		Blank Answers	
	f	%	f	%	f	%
1	138	69	60	30	2	1
2	110	55	89	44,5	1	0,5
3	100	50	94	47	6	3
4	71	35,5	126	63	3	1,5
5	110	55	84	42	6	3
14	67	33,5	72	36	61	30,5
15	41	20,5	87	43,5	72	36
16	98	48	30	15	72	36
17	29	14,5	91	45,5	80	40
21	34	17	59	29,5	107	53,5

Table 5 shows that 39.9% of students answered the questions correctly, 39.6% answered incorrectly, and 20.5% left the questions blank. Among the questions asked, Question 4 was answered incorrectly most frequently (63%), while Question 1 was answered correctly most frequently (69%).

It was determined that students had misconceptions in the types of overgeneralization, overspecification, mistranslation, and limited perception when naming polygons, creating them, and identifying their basic elements. It was determined that students with overgeneralization misconceptions stated that the letter sequence should be alphabetical when naming polygons, thought that a diagonal was any straight-line segment within a polygon, named polygons with parallel sides as parallelograms, and classified a wide-angled triangle as an acute-angled triangle because of its two narrow angles. The sample answers provided by students in the context of their belief that the letter sequence should be alphabetical when naming polygons are given below:

"I think it's called ABCD." (S6), "Because it doesn't go ABDC." (S53)

Below are the sample answers given by students in the context of thinking that a diagonal is any line segment within a polygon:

"Because it's a rectangle, and what remains inside the rectangle is the diagonal." (S161)

The sample answers given by students regarding naming polygons with parallel sides as parallelograms are given below:

From the answers given to Question 4: *"The sides are parallel to each other." (S11), "In ABCD, AD is parallel, BC is parallel." (S41)*

The sample answers given by students in the context of thinking that diagonals are fundamental elements in polygons are given below:

"Without a vertex, a side, and a diagonal, there can be no square, rectangle, or triangle." (S75)

Examples of students' responses in the context of classifying a wide-angled triangle as an acute-angled triangle due to its two narrow angles are given below:

"The first triangle is an acute-angled triangle, and only Figure 1 in the visual is an acute-angled triangle." (S53)

Students were found to have misconceptions about excessive generalization, such as not including triangles in the polygon group, classifying obtuse triangles as acute triangles because of their two narrow angles, and thinking that the property of parallel sides belongs only to parallelograms. Students' responses regarding these findings are given below:

Responses to Question 1: *"1 and 4 are not possible; a triangle is not a polygon; it must have at least 4 corners." (S91)*

Responses to Question 15: *"There are no wide-angled triangles." (S27)*

"Because it says parallel, it is a parallelogram." (S24), "Because it says parallel." (S35)

Students exhibit misconceptions in the form of incorrect translation, demonstrating shape perception disorders in polygons, confusing the concept of "trapezoid" with "curve," mixing up the definition of "corner" with "sharp point" and "edge," and confusing the concepts of 'edge' and "corner." use letter sequences when naming polygons, confuse the concepts of "diagonal" and "internal angle," confuse the terms 'corner' and "diagonal" due to their similar names, they thought that the diagonal was an interior angle because it connected opposite corners, they thought that each side of a trapezoid had a different length, they classified a wide-angled triangle as a narrow-angled triangle because of its two narrow angles, they confused the definition of "parallelism," and they thought that any shape containing a diagonal could be converted into a triangle. An example response given by students indicating a distorted perception of polygons:

From the answers given to Question 1: *"1, 3, 5 are not polygons." (S14)*

An example response given by students in the context of confusing the concept of a trapezoid with a curve:

From the answers given to Question 1: *"Because the first picture is a trapezoid." (S107)*

Example response given by students in the context of confusion in the definition of the term "corner":

"The place where two sides meet is called a point." (S112)

Example responses given by students in the context of confusing the concepts of side and vertex:

"It can be a vertex or it can be a side." (S147), From the responses given to Question 2: "A side is called the outer end." (S169)

Example answers given by students in the context of using letters when naming polygons:

"I called it polygon A because that's what it was written as." (S46), From the answers given to Question 2: *"Because it's a square polygon." (S139)*

Examples of students' responses in the context of confusing the concepts of diagonal and interior angle:

From the responses to Question 3: *"It's an interior angle because it passes through the interior." (S72), "Opposite sides are interior angles." (S178)*

Example answers given by students in the context of confusing the terms vertex and diagonal due to their similarity in name:

From the answers given by students to Question 3: *"It shows the A-C vertices." (S39), "Because it goes towards the vertex." (S89)*

Students' responses in the context of thinking that the diagonal is an interior angle because it connects opposite corners:

"Opposite sides are interior angles." (S70) The example response also shows that the distinction between the terms 'corner' and "side" has not been made.

Responses given in the context of students thinking that each side of a trapezoid has a different length:

"If it were a trapezoid, each side would be different." (S132).

Responses given in the context of students naming the acute triangle as an obtuse triangle because of its two narrow angles:

From the responses given to Question 15: *"The first triangle is an obtuse triangle, and only Figure 1 in the visual is an obtuse triangle." (S38).*

Students' responses in the context of having shape perception disorders in polygons:

From the responses given to Question 16: *"Because the others are not triangles." (S97) (Option C was marked.)*

Responses given by students in the context of confusion about the definition of parallelism:

"Two sides of the triangle are parallel." (S107).

Example responses given in the context of students thinking that any quadrilateral with a diagonal can be converted into a triangle:

"Because it becomes a triangle from its interior angles." (S111) From the responses given to Question 21.

Students' responses in the context of confusing the concepts of diagonal and interior angle:

"It's an interior angle because the line passes through the interior." (s143)

Students were found to have a limited perception misconception type, with shape perception disorders in polygons, classifying a wide-angled triangle as a narrow-angled triangle due to its two narrow angles. Example responses given by students in the context of shape perception disorder in polygons:

From responses given to Question 1: *"Because the first picture is a trapezoid." (S12),* From responses given to Question 16: *"Because the others are not triangles." (S152).*

Examples of students' responses in the context of classifying the wide-angled triangle as an acute-angled triangle due to its two narrow angles:

From the responses given to Question 15: *"The first triangle is an acute-angled triangle, and only Figure 1 in the visual is an acute-angled triangle." (S126)*

Misconceptions of 5th grade middle school students regarding forming triangles based on angles and sides, and classifying different triangles based on side and angle properties

Five questions were asked to fifth-grade students to identify their misconceptions regarding forming triangles based on angles and sides and classifying different triangles according to their side and angle properties. The students correct, incorrect, and blank answers to these questions are presented in Table 6.

Table 6. Distribution of Students' Answers to Questions

Questions	Correct Answer		Incorrect Answers		Blank Answers	
	f	%	f	%	f	%
6	139	69,5	55	27,5	6	3
7	169	84,5	21	10,5	10	5
11	117	58,5	54	27	29	14,5
19	54	27	55	27,5	91	45,1
20	54	27	47	23,5	99	49,5

Table 6 shows that 53.3% of students answered the questions correctly, 23.2% answered incorrectly, and 24.5% left the questions blank. Among the questions, Question 7 was answered correctly by 84.5% of students, while Question 6 and Question 19 were answered incorrectly by 27.5%.

It was determined that students had misconceptions of the types of overgeneralization and restricted perception regarding forming triangles based on angles and sides and classifying different formed triangles according to side and angle properties. In terms of over-specification misconceptions, it was determined that students thought a triangle with two acute angles was an acute-angled triangle because it had two acute angles, and that they had a prototype shape misconception about triangles. The following are examples of students' responses in the context of thinking that a triangle with two acute angles is an acute-angled triangle because it has two acute angles:

Responses to Question 6: "*Because it is an acute-angled triangle.*" *(S13)*, "*Because angle ABC is acute.*" *(S29)*; Responses to S20: "*Because the triangle is acute and its sides are of different lengths.*" *(S150)*.

Examples of responses given by students in the context of prototypical shape misconceptions in triangles:

From the responses to Question 19: "*It is the least triangular of them.*" *(S134)*.

It was determined that students had a misconception of incorrect translation, thinking that a triangle with two acute angles is an acute-angled triangle because it has two acute angles, and that any triangle with a perpendicular line drawn from one corner to the opposite side is a right-angled triangle. Example responses given in the context of students thinking that an obtuse triangle is an acute triangle because two of its angles are acute:

From the responses given to Question 6: "*Because it is an acute-angled triangle.*" *(S13)*, "*Because angle ABC is acute.*" *(S29)*; From the responses given to S20: "*Because the triangle is acute and its sides are of different lengths.*" *(S150)*.

Example answers given in the context of students thinking that a perpendicular line drawn from any corner of a right-angled triangle to the opposite side forms a right-angled triangle:

From the answers given to Question 11: "*Because it forms 90° when it meets point E.*" *(S49)*

Misconceptions of 5th grade students regarding identifying and drawing the basic elements of rectangles, parallelograms, rhombuses, and trapezoids

Five questions were asked to 5th grade students to identify their misconceptions regarding identifying and drawing the basic elements of rectangles, parallelograms, rhombuses, and trapezoids. The students' answers to these questions are presented in Table 7 as correct, incorrect, or blank.

Table 7. Distribution of Students' Answers to Questions

Questions	Correct Answer		Incorrect Answers		Blank Answers	
	f	%	f	%	f	%

8	121	60,5	67	33,5	12	6
9	79	39,5	98	49	27	13,5
10	87	43,5	93	46,5	20	10
18	52	26	57	28,5	109	54,5
22	30	15	54	27	116	58

According to Table 7, 36.9% of students answered the questions correctly, 36.9% answered incorrectly, and 26.2% left them blank. Among the questions, Question 8 had the highest percentage of correct answers at 60.5%, while Question 9 had the highest percentage of incorrect answers at 49%.

It was determined that 5th grade students had misconceptions related to identifying and drawing the basic elements of rectangles, parallelograms, rhombuses, and trapezoids, including overgeneralization, mistranslation, and limited perception. In terms of overgeneralization misconceptions, they thought that all shapes with equal opposite sides were parallelograms, they had a prototype shape misconception specifically for trapezoids and parallelograms, they classified all quadrilaterals as rectangles, and they equated the situation where the lengths of the opposite sides of a polygon are equal with the situation where all side lengths are equal. Example answers given by students to questions in the context of thinking that all shapes with equal opposite sides are parallelograms:

Responses to Question 8: "*Everything on the opposite side that is the same length is parallel.*" *(S159).*

Examples of students' responses to questions in the context of prototype shape misconceptions specifically regarding trapezoids and parallelograms:

From the responses to Question 9: "*The shape resembles a prism.*" *(S52).* From the example responses to Question 10: "*Because option D is also a parallelogram.*" *(S103)*

Examples of students' responses to questions in the context of classifying all quadrilaterals as rectangles:

From the responses given to Question 9: "*Number one cannot be considered a rectangle.*" *(S61)*, "*2 and 1, 5 and 4 are the same type, leaving 3.*" *(S74).*

Examples of students' responses to questions in the context of equating the equality of opposite sides of a polygon with the equality of all sides:

From the responses to Question 22: "*Because the opposite sides are the same.*" *(S98) (Option B is marked).*

It was determined that students had misconceptions of the type of incorrect translation, confusing parallelism with the situation where the side lengths are equal, thinking that the opposite sides of a parallelogram are not equal in length, not knowing the definition of a parallelogram, and thinking that the diagonal is a determining element of the interior angles in a polygon. Example answers given by students in the context of confusing parallelism with the condition of equal side lengths:

From the answers given to Question 8: "*Because they are equal and twins.*" *(S46) (Option D was marked).*

Example answers given in the context of students thinking that opposite sides of a parallelogram are not equal:

From the responses given to Question 18: "*Because the opposite sides are not equal.*" *(S175) (Option D was marked)*

Examples of responses given by students in the context of not knowing the definition of a parallelogram:

"*Because a parallelogram does not have 4 corners.*" *(S60).*

Example answers given by students in the context of thinking that the diagonal is a determining element of the interior angles in a polygon:

From the answers given to Question 22: "*The interior angles of the diagonals may not be the same.*" *(S116) (Option D was marked)*

It was determined that students have a limited perception misconception, thinking of any polygon with equal sides as an equilateral triangle and having a prototype shape misconception specifically for rhombuses and parallelograms. Example responses given by students in the context of thinking that any polygon with equal sides is an equilateral triangle:

"Equilateral means all sides are equal." (S109) (Option D was marked).

Examples of responses given by students in the context of the prototype shape misconception regarding trapezoids and parallelograms:

From the responses given to Question 9: "*The shape resembles a prism." (S52).* From the example responses given to Question 10: "*Because option D is also a parallelogram." (S103).*

Misconceptions of 5th grade students regarding determining the sum of the interior angles of triangles and quadrilaterals and finding the unknown angle

Four questions were asked to 5th grade students to identify their misconceptions regarding determining the sum of the interior angles of triangles and quadrilaterals and finding the unknown angle. The students' responses to these questions are presented in Table 8.

Table 8. Distribution of Students' Answers to Questions

Questions	Correct Answer		Incorrect Answers		Blank Answers	
	f	%	f	%	f	%
12	67	33,5	93	46,5	40	20
13	59	29,5	73	36,5	68	34
23	34	17	56	28	110	55
24	18	9	63	31,5	119	59,5

According to Table 8, 22.25% of students answered the questions correctly, 35.625% answered incorrectly, and 42.125% left them blank. The question with the highest percentage of correct answers was Question 12 at 33.5%, while the question with the highest percentage of incorrect answers was again Question 12 at 46.5%.

It was determined that students had a limited type of misconception regarding determining the sum of the interior angles of triangles and quadrilaterals and finding the unknown angle. In this type of misconception, it was found that students thought the sum of the interior angles of a triangle was equal to the sum of the angles given in the figure and could not define the situation where any line drawn from a vertex of a triangle to the opposite side divides the triangle into two different triangles. Example answers given in the context of students thinking that the sum of the interior angles of a triangle is equal to the sum of the angles given in the figure:

From the responses to Question 13: "*Because two corners of the triangle were given, I added them up and got 180." (S42)*. From the responses to Question 23: "*30+30=60" (S64).*

Example responses given in the context of students' inability to define the situation where any line drawn from one corner of a triangle to the opposite side divides the triangle into two different triangles:

From the responses given to Question 13: "*I added 102 and 62 and subtracted the result from 180." (S107).*

Conclusion and discussion

When examining the misconceptions of 5th grade students regarding naming polygons, creating them, and identifying their basic elements, it was found that shapes consisting of non-straight sides but having corners were accepted as polygons or were considered trapezoids. Therefore, it was concluded that there is a distortion in the perception of polygon shapes. Students do not include triangles in the polygon group and treat them separately. Similar to the research results published by Başışık (2010), this situation is partially observed, and it is stated that equilateral triangles are included in the polygon class, while different triangles are not included in the polygon class based on their angles and sides. Some students, however, believe that inverted triangles or triangles positioned differently are not triangles due to their prototypical shape habit. Misconceptions regarding prototypical shape perception are similar to the results of Uygun's (2023) study. Furthermore, when classifying triangles according to their angles, students base their classification on all the angles within the triangle. A common misconception is that a triangle with a wide angle is a triangle with two narrow angles because it appears narrow visually. Baran (2011) states that students have conceptual confusion about acute-angled and obtuse-angled triangles, as well as conceptual misconceptions about isosceles and equilateral triangles.The fact that students classify triangles in inverted or unusual positions as "not triangles" corresponds exactly to the phenomenon described in the literature as the "Prototype Effect" (Hershkowitz, 1990). As Clements (1999) also noted, students exclude variations that do not conform to the ideal prototype image in their minds (typically triangles with bases parallel to the ground). Furthermore, the failure to include triangles in the polygon group or the belief that only equilateral triangles belong to this class highlights students' deficiencies in hierarchical classification skills. This finding aligns with De Villiers (1994)'s "exclusive classification" approach and demonstrates that students cannot conceptualize the logical structure where a shape can belong to more than one category simultaneously.

It is seen that many students experience conceptual confusion regarding the definition of the term "corner." While some students define the term "corner" as a "sharp point," others cannot distinguish between the term's 'side' and "corner." The fact that students define the term "corner" as a "sharp point" or fail to distinguish between the concepts of 'edge' and "corner" is a result of the conflict between everyday language and mathematical language, as emphasized by Pimm (1987). Confusing the concept of a diagonal with an interior angle or believing that a diagonal is an indispensable element of a polygon, indicates that students have a limited level of understanding regarding these terms. Similarly, classifying a wide-angled triangle as "narrow-angled" simply because it appears narrow visually supports Tall and Vinner (1981)'s theory of inconsistency between "conceptual image" and "conceptual definition." The dominant visual image in the student's mind overrides the academic definition, leading to an erroneous conclusion.

The misconceptions students have specifically regarding the concept of "diagonal" are: confusing the diagonal with the concept of interior angle because it passes through the polygon, believing that the diagonal is an essential element in polygons, and experiencing conceptual confusion based on the similarity in names between the corner and the diagonal. The study by Duatepe-Paksu, İymen and Pakmak (2013) concluded that the concept of a diagonal is perceived as a vertex, and the researchers stated that the main reason for this is the association of the diagonal with the interior angle. Some students defined the diagonal as any line passing through the polygon. However, this definition is too general and also includes a line segment starting from the sides rather than the corners. This situation will not meet the definition of the concept of a diagonal and will therefore create a misconception. Another misconception found regarding diagonals is that students think quadrilaterals with diagonals can be divided into two separate triangles.

The study concluded that students experience many misconceptions regarding naming, constructing, and identifying the basic elements of polygons. While some students believe that polygons with at least two parallel sides are parallelograms and that this feature is unique to parallelograms, others believe that each side of a trapezoid is of a different length, which is similar to the findings of Başışık's (2010) research.

When examining the misconceptions of 5th grade middle school students regarding forming triangles based on their angles and sides and classifying different triangles according to their side and angle properties, it is seen that most students have misconceptions regarding the prototypical shape of triangles and the classification of triangles based on their sides. They believe that an obtuse triangle should be called an acute triangle because it has acute angles, and that a right triangle is defined not by the perpendicularity of its sides but by the perpendicular line (height) drawn from the vertex to the opposite side. The tendency identified in the study for students to classify obtuse triangles as acute triangles strikingly highlights the inconsistency between the "conceptual image" and the "conceptual definition" described by Tall and Vinner (1981). Rather than considering the definitional criteria of the shape (a wide angle), students focus on the visually dominant image of a narrow angle. Similarly, the fact that the definition of a right triangle is based on the concept of height (perpendicular) rather than side ratios aligns with the issue of limited perception regarding the concept of height highlighted by Gutierrez and Jaime (1999). These findings confirm that students' reliance on prototypical images in geometry (Hershkowitz, 1990) is a fundamental factor hindering the proper construction of scientific definitions.

When examining the conceptual misconceptions of 5th grade middle school students regarding identifying and drawing the basic elements of rectangles, parallelograms, rhombuses, and trapezoids, it was found that some students had a prototype shape misconception about trapezoids and did not consider a right trapezoid to be a trapezoid. This situation is consistent with the research results of Başışık (2010) and Doğan et al. (2012).

Regarding quadrilaterals, it was observed that some students believed that all quadrilaterals were rectangles. In addition, it was observed that they had a misconception about the difference between the lengths of opposite sides being equal and all sides being equal in length. Again, based on the students' responses, a misconception was identified regarding the diagonals being a determining factor for the interior angles. Duatepe-Paksu, İymen, & Pakmak (2013) state that diagonals are associated with interior angles and are therefore confused with interior angles or corner concepts.

When examining the misconceptions of 5th grade middle school students regarding determining the sum of the interior angles of triangles and quadrilaterals and finding the unknown angle, it was found that students have a misconception that all angles of an isosceles triangle are equal, which is a property of an equilateral triangle, and that the sum of the interior angles of a triangle is equal to the sum of the given angles. Furthermore, it was determined that some students had misconceptions regarding the representation of angles with symbols and the representation of triangles with symbols.

Based on these findings, the following recommendations can be made:

- Considering that students confuse the basic concepts of triangles and quadrilaterals, it is thought that using concrete materials and dynamic geometry software in teaching these topics would be useful in eliminating and preventing possible misconceptions.
- Considering the finding that students have a standard image of polygons in their minds and generally have an incorrect prototype shape perception, it is thought that teachers diversifying geometric shapes in different sizes and positions in classroom activities would be useful in eliminating and preventing misconceptions on this subject.
- Considering the misconception identified regarding students' confusion between the concepts of trapezoid and curve, it is thought that mathematical language should be used more by teachers in classroom activities and discussions, especially in mathematics lessons. This can be said to be important in order to support students in distinguishing between everyday language and mathematical language.
- It has been determined that students confuse certain mathematical rules expressed by teachers in teaching about triangles and quadrilaterals. This may be related to teachers teaching based on “procedural knowledge.” Therefore, to prevent such misconceptions from arising in students, teaching triangles and quadrilaterals should support students in discovering the rules by using materials that allow them to practice with shapes and rules.
- This study identified the misconceptions of 5th grade middle school students regarding triangles and quadrilaterals. In future studies, teaching activities aimed at eliminating these identified misconceptions can be designed and studies can be conducted using experimental or action research designs.
- Researchers can design qualitative studies based on clinical interviews to identify students' misconceptions about triangles and quadrilaterals, with the aim of thoroughly examining the logical and mental connections students make when answering questions and any possible gaps in their prior knowledge.

Declarations

Ethics statements

This study complies with all the rules specified in the Higher Education Institutions Scientific Research and Publication Ethics Guidelines. Ethical committee approval for this research was obtained from the Ordu University Education Research Ethics Committee with its decision dated 28/02/2025 and numbered 2025-31. All individuals participating in the study were informed about the purpose and scope of the research, and the “Informed Consent Form” was signed after ensuring the confidentiality and anonymity of the data.

Informed consent

All individuals participating in the study were informed about the purpose and scope of the research, and the “Informed Consent Form” was signed after ensuring the confidentiality and anonymity of the data.

Availability of data and materials

Due to ethical considerations and participant confidentiality, the qualitative data is not publicly available but may be obtained from the corresponding author upon reasonable request.

Competing interests

All financial and non-financial competing interests must be declared in this section.

Funding

This research received no external funding.

Authors' contributions

Author 1: Conceptualization; Methodology; Investigation; Data curation; Data analysis; Writing—original draft; Writing—review & editing. Author 2: Investigation; Resources; Data curation; Data analysis; Visualization; Writing—review & editing. All authors have read and approved the final version of the manuscript.

Artificial Intelligence

Only AI-powered tools were used in the language editing process of this article. No AI-powered tools were used in any other section of the article.

Acknowledgments

Our article titled " Identifying Misconceptions of 5th Grade Students Regarding Triangles and Quadrilaterals" has no financial conflict of interest with any institution, organization or person.

References

Abbak, Y. & Gelişli, Y. (2023). Scope validity study of the outcomes of the in-service training program to be prepared for teaching 21st century skills according to Davis technique. *Millî Eğitim Journal, 52* (Özel Sayı), 35-56. https://doi.org/10.37669/milliegitim.1309227

Altun, M. (2018). *Ortaokullarda matematik öğretimi [Mathematics instruction in middle schools].* Aktüel Yayınları.

Araz, S. (2024). *7. sınıf matematik dersi rasyonel sayılar konusundaki kavram yanılgılarının matematiksel modelleme etkinlikleri ile yapılan öğretim bağlamında incelenmesi [An examination of misconceptions regarding rational numbers in 7th grade mathematics lesson within the context of teaching with mathematical modeling activities].* (Thesis Nu:898419) [Master's Thesis, Van Yüzüncü Yıl University]. Turkish Council of Higher Education Thesis Center.

Ay, Y. & Başbay, A., (2017). Misconceptions about polygons and their possible reasons. *Ege Journal of Education, 18*(1), 83-104. https://doi.org/10.12984/egeefd.328377

Baran, S. (2011). *İlköğretim II. Kademe öğrencilerinin üçgenler ve geometrik cisimler konusundaki kavram yanılgıları [The misconceptions of the second-grade students about the triangles and geometric objects].* (Thesis Nu:285503) [Master's Thesis, Van Yüzüncü Yıl University]. Turkish Council of Higher Education Thesis Center.

Başışık, H., (2010), *İlköğretim 5. Sınıf öğrencilerinin çokgenler ve dörtgenler konularındaki kavram yanılgılarının belirlenmesi [Determining fifth grade students' misconceptions on polygons and quadrilaterals].* (Thesis Nu:293041) [Master's Thesis, Van Yüzüncü Yıl University]. Turkish Council of Higher Education Thesis Center.

Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). *Rational-Number concepts*. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 91-126). New York, NY: Academic Press.

Booth, L. R. (1988). *Children's difficulties in beginning algebra*. In A.F. Coxford & A. P. Schulte (Eds.), The ideas of algebra, K-12, 1988 Yearbook, (pp. 20-32), Reston, Va.: NCTM.

Chen, C.C., Lin, H.S., & Lin, M.L. (2002). Developing a two-tier diagnostic instrument to assess high school students' understanding-the formation of images by a plane mirror. *Proceedings of the National Science Council, 12*(3), 106-121.

Clements, D. H. (1999). Geometric and spatial thinking in young children. In J. V. Copley (Ed.), *Mathematics in the Early Years* (s. 66-79). Reston, VA: National Council of Teachers of Mathematics & Washington D.C.: NAEYC.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 420–464). Macmillan Publishing Co, Inc.

Çekiç, E. (2018). *Ortaokul 5. Sınıf öğrencilerinin temel geometrik kavramlar ve çizimler alt öğrenme alanına yönelik kavram yanılgıları [Misconceptions of fifth graders regarding the basic geometric concepts and drawings sublearning domain].* (Thesis Nu:542110) [Master's Thesis, Muğla Sıtkı Koçman University]. Turkish Council of Higher Education Thesis Center.

Dağ, Ş. (2022). *Matematik eğitiminde kavram yanılgıları konusunda yapılmış araştırmaların incelenmesi: Sistematik derleme çalışması [Investigation of studies on misconceptions in the mathematics education: Systematic review study].* (Thesis Nu:752444) [Master's Thesis, Necmettin Erbakan University]. Turkish Council of Higher Education Thesis Center.

De Villiers, M. (1994). The role and function of a hierarchical classification of quadrilaterals. *For the Learning of Mathematics, 14*(1), 11-18.

Doğan, A., Özkan, K., Çakır, N. K., Baysal, D. ve Gün, P. (2012). Students' misconceptions about trapezium through primary levels. *Uşak University Journal of Social Sciences, 5*(1), 104- 116. https://doi.org/10.12780/UUSBD108

Duatepe-Paksu, A., İymen, E. ve Pakmak, G. S. (2013). Pre-service elementary teachers' concept images on diagonal of quadrilaterals. *Education and Sciences*, 38(167), 162-178. https://doi.org/10.15390/ES.2013.1109

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics*, *24*(2), 139-162.

Gökdoğan, S. (2021). *Kavram yanılgılarını belirlemede yapılandırılmış grid ve üç aşamalı test yöntemlerinin karşılaştırılması [Comparison of configured grid and three-stage test methods in determining misconceptions].* (Thesis Nu:694584) [Master's Thesis, Hacettepe University]. Turkish Council of Higher Education Thesis Center.

Gutiérrez, A., Jaime, A. (1999). Preservice primary teachers' understanding of the concept of altitude of a triangle. *Journal of Mathematics Teacher Education* 2, 253–275. https://doi.org/10.1023/A:1009900719800

Hacısalihoğlu-Karadeniz, M. & Hodancı, A. N. (2022). Determining sixth grade students' misconceptions about integers. *Journal of Computer and Education Research*, *10*(20), 358-370. https://doi.org/10.18009/jcer.1087195

Hershkowitz, R. (1990). *Psychological aspects of learning geometry*. In Mathematics and Cognition. Cambridge University Press.

Hiebert, J., & Carpenter, T.P. (1992). *Learning and teaching with under- standing*. In: D. A. Grouns (Ed.), Handbook of research on mathematics teaching and learning (pp. 65-92). New York: Macmillan.

Kabadaş, H. (2022). *Ortaokul matematik öğretmenlerinin öğrencilerin cebirdeki muhtemel kavram yanılgılarını öngörme ve giderme süreçlerinin incelenmesi [Investigation of secondary school mathematics teachers' prediction and elimination processes of students' possible misconceptions in algebra].* (Thesis Nu:753719) [Master's Thesis, Ordu University]. Turkish Council of Higher Education Thesis Center.

Kaya, N. (2018). *Ortaokul sekizinci sınıf öğrencilerinin üçgenler alt öğrenme alanındaki kavram yanılgılarının incelenmesi [An Examination of eighth grade middle school students' misconceptions about triangles].* (Thesis Nu:526840) [Master's Thesis, İnönü University]. Turkish Council of Higher Education Thesis Center.

Kemankaşlı, N. ve Gür, H. (2005). An analysis of errors made by secondary school students in the "quadrilaterals" unit of their geometry course. *Proceedings of the* XIV. National Congress of Educational Sciences, Pamukkale University, Denizli.

Ministry of National Education. (2018). *Mathematics curricula*. http://mufredat.meb.gov.tr/ProgramDetay.aspx?PID=329

Öksüz, C. ve Başışık, H. (2019). Determining the misconceptions of fifth grade students about polygons and quadrilaterals. *Eskişehir Osmangazi University Journal of Social Sciences, 20*, 413-430. https://doi.org/10.17494/ogusbd.548525

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. Routledge.

Sarı, F. (2024). *Altıncı sınıf öğrencilerinin ondalık gösterimlerde oluşan kavram yanılgılarının matematiksel modelleme yoluyla giderilmesinin incelenmesi [Investigating the elimination of sixth-grade students' misconceptions in decimals through mathematical modeling].* (Thesis Nu:863554) [Master's Thesis, Zonguldak Bülent Ecevit University]. Turkish Council of Higher Education Thesis Center.

Smith, J. P., diSessa, A. A., & Roschelle, J. (1993-1994). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *Journal of the Learning Sciences*, *3*(2), 115–163. https://doi.org/10.1207/s15327809jls0302_1

Şahiner, F. (2018). *Ortaokul 8. sınıf öğrencilerinin matematik dersi cebirsel ifadeler konusundaki kavram yanılgıları [Misconceptions on algebraic exppressions in maths lesson for secondary school 8th grade students].* (Thesis Nu:512922) [Master's Thesis, Akdeniz University]. Turkish Council of Higher Education Thesis Center.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics. *Educational Studies in Mathematics*, *12*(2), 151-169. https://doi.org/10.1007/BF00305619

Toraman, S. (2021). Mixed methods research: A Brief history, definitions, perspectives, and key elements. *Qualitative Social Sciences*, *3*(1), 1-29. https://doi.org/10.47105/nsb.847688

Treagust, D. F. (1988). Development and use of diagnostic tests to evaluate students' misconceptions in science. *International Journal of Science Education*, *10*(2), 159-169. https://doi.org/10.1080/0950069880100204

Türkdoğan, A., Güler, M., Bülbül, B., & Danişman, Ş. (2015). Studies about misconceptions in mathematics education in Turkey: A Thematic review. *Mersin University Journal of the Faculty of Education*, *11*(2), 215-236. https://doi.org/10.17860/efd.26545

Uygun, S. (2023). *Matematik öğretmenlerinin ortaokul geometri konularında yaşadıkları hata ve kavram yanılgıları [Mistakes and misconceptions of mathematics teachers in secondary school geometry].* (Thesis Nu:780732) [Master's Thesis, Kafkas University]. Turkish Council of Higher Education Thesis Center.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction, 14*(5), 453–467. https://doi.org/10.1016/j.learninstruc.2004.06.013

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. London: Academic Press.

Vinner, S. (1991). *The role of definitions in the teaching and learning of mathematics*. In Tall, D. (Ed.), Advanced Mathematical Thinking (pp. 65-81). Boston: Kluwer.

Vosniadou, S. (1994). Capturing and modeling the process of conceptual change. *Learning and Instruction,* 4(1), 45-69. https://doi.org/10.1016/0959-4752(94)90018-3

Yurtyapan, M. İ., & Karataş, İ. (2020). An investigation of pedagogical content knowledge of secondary school mathematics teachers in relation with triangle and quadrilateral concepts. *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 11*(1), 53-90. https://doi.org/10.16949/turkbilmat.443825

Zembat, I. O. (2008). Kavram yanılgısı nedir? [What Is a Misconception?] In M. F. Ozmantar, E. Bingolbali, & H. Akkoc (Eds.), *Matematiksel Kavram Yanılgıları ve Çözüm Önerileri [Mathematical Misconceptions and Suggested Solutions]* (pp. 1-8). Ankara: Pegem Akademi.

Zembat, İ. Ö. (2013). Sayıların farklı algılanması-sorun sayılarda mı, öğrencilerde mi? [Different perceptions of numbers—Is the problem with the numbers or with the students?] In M. F. Özmantar, E. Bingölbali, & H. Akkoç (Eds.), *Matematiksel Kavram Yanılgıları ve Çözüm Önerileri [Mathematical Misconceptions and Suggested Solutions]* (s. 41-60). Ankara: Pegem Akademi.

Zodik, I., & Zaslavsky, O. (2008). Characteristics of teachers' choice of examples in and for the mathematics classroom. *Educational Studies in Mathematics, 69*(2), 165–182. http://www.jstor.org/stable/40284541